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Is the Black Hole Complementarity principle really necessary?

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Abstract

We show that the S-matrix ansatz implies a semi-classical metric such that a freely falling test particle will not cross the horizon in its proper time. Instead of reaching the singularity it will reach $\mathcal{I}^+$.

One of the most interesting questions that one has to confront if one adopts 't Hooft S-matrix ansatz [1] is whether or not a freely falling observer can cross the horizon in his way to the singularity. On the one hand, the curvature is small at the horizon, of the order of $\frac{1}{M^2}$ [2], and at least classically there is nothing special at the horizon for a freely falling observer. Furthermore, a freely falling observer cannot detect the Hawking temperature since for such an observer the Hawking radiation is part of the vacuum fluctuations. On the other hand, according to the S-matrix ansatz the information is encoded in the Hawking radiation due to strong gravitational interactions that take place just outside the horizon.

This conflict lead Susskind, Thorlacius and Uglum to suggest the black hole complementarity principle [2] which can be formulated as follows:

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[2]In units where $G = \hbar = c = 1$.

- From the point of view of an external observer the region just outside the horizon (stretched horizon) acts like a very hot membrane which absorbs thermalizes and emits any information that falls to the black hole.
- From the point of view of a freely falling observer there is nothing special at the horizon so a freely falling observer can cross the horizon in his way to the singularity.

At first sight, this principle seems to be inconsistent. But a more careful analysis of some Gedanken experiments shows that the black hole complementarity principle may be consistent [2].

In this letter we do not consider the consistency of the black hole complementarity principle but we show that it is not necessary, namely that the S-matrix ansatz is complete by itself. In particular, we show that the S-matrix ansatz implies such a gravitational back-reaction that a freely falling observer will not cross the horizon even in his proper time. This implies that the Penrose diagram describing gravitational collapse has the same topology as Minkowski space. Such a Penrose diagram was already suggested in [3, 4] using the weak value [5] of the metric as defined by an external observer. Our point is that we do not consider the weak value but the mean value as implied by the S-matrix ansatz. Therefore our results hold also for an infalling observer.

Classically, a freely falling observer can cross the horizon in a finite amount of proper time. Quantum mechanically, the black hole radiates so a freely falling observer is moving along a geodesic in the metric induced by the black hole and the Hawking radiation. In order to find the geodesics

one should be able to calculate the mean energy-momentum tensor of the radiation, $T_{\mu\nu}^{rad}$, and to take it as a source to the Einstein's equations in a Schwarzschild background. However, in field theory $T_{\mu\nu}^{rad}$ diverges and should be renormalized. Unfortunately, the Einstein-Hilbert Lagrangian is non-renormalizable. Nevertheless, under some "reasonable" assumptions one can regularize and renormalize $T_{\mu\nu}^{rad}$ where for simplicity the star in is taken to be a thin spherically symmetric shell and the centrifugal barrier is neglected (only the s-wave sector is taken into account) [6]. The "reasonable" assumption is that the metric and $T_{\mu\nu}^{rad}$ are regular for a freely falling observer. Naturally, this assumption leads to a metric which allows a freely falling observer to cross the horizon. The mean energy-momentum tensor of a spin-less massless field that was found in [6] is

$$\begin{aligned} 4\pi r^2 T_{uu}^{rad} &= \frac{1}{12\pi}\left[-\frac{M}{2r^3}(1-\frac{2M}{r})-\frac{M^2}{4r^4}\right]+\frac{\pi}{12(8\pi M)^2} \\ 4\pi r^2 T_{vv}^{rad} &= \frac{1}{12\pi}\left[-\frac{M}{2r^3}(1-\frac{2M}{r})-\frac{M^2}{4r^4}\right] \\ 4\pi r^2 T_{uv}^{rad} &= -\frac{M}{12\pi 2r^3}(1-\frac{2M}{r}), \end{aligned} \tag{1}$$

The physical origin of this energy-momentum tensor is that pairs of "particles" are created in the region above the horizon one with negative energy which falls into the black hole, and one with positive energy which reaches $\mathcal{I}^+$. Note that for any r, $4\pi r^2(T_{uu}-T_{vv}) = \frac{\pi}{12(8\pi M)^2}$ which is the thermal flux of the Hawking radiation, up to a numerical constant due to the fact that the centrifugal barrier was dropped.

Notice further that near the horizon the mass evaporation is due to a negative T_{vv}^{rad} and not to a positive T_{uu}^{rad} (at $r = 2M$ one gets $T_{uu}^{rad} = 0$) .

Thus in this scenario at least semi-classically there are no strong gravitational interactions near the horizon supporting Hawking's picture [7] . The reason is that in this scenario near the horizon all the energy momentum flux is in the v direction . Namely, T_{vv} is positive for ingoing light particles and negative for Hawking radiation and all other components of $T_{\mu\nu}$ vanish, so $T^{rad}_{\mu\nu}$ and $T^{in}_{\mu\nu}$ are parallel hence they do not interact. As a result there is an *exact* solution to Einstein's equation near the horizon of a black hole which is constructed out of ingoing light like flux including the back reaction of Hawking particles. The exact solution can be expressed by the ingoing Eddington-Finkelstein coordinate (the Vaidya solution):

$$ds^2 = -(1 - \frac{2M(v)}{r})dv^2 + 2dvdr + r^2 d\Omega^2 \tag{2}$$

where

$$M(v) = \int^v dv^{'} (T^{rad}_{v^{'}v^{'}} + T^{in}_{v^{'}v^{'}}). \tag{3}$$

The ingoing null geodesics in this metric are $v = const$, so it is clear that in this scenario a freely falling observer can cross the horizon (for a recent review see [10]). But it is important to emphasis that in fact that result was assumed by the regularity condition at the horizon.

This scenario cannot coexist with the S-matrix ansatz. The reason is that if the information is encoded in Hawking radiation then there should be strong interactions between ingoing and outgoing particles. Causality implies that interactions taking place behind the horizon will not affect the final state of the radiation and interactions which occur at large distances from the horizon are between regular particles (no Planckian energies) so their effect

is too weak. Therefore the S-matrix ansatz implies that strong interactions should take place just outside the horizon unlike Hawking's scenario.

In [1] an approximation to the S-matrix was suggested. The approximation is based on the classical gravitational interaction between two light particles (the gravitational shock wave) and the WKB approximation. The gravitational field of a massless particle in Minkowski space is described by the line element

$$ds^2 = -dU(dV + 4p\ln(\frac{\tilde{x}^2}{M^2})\delta(U-U_0)dU) + dx^2 + dy^2 \tag{4}$$

where $\tilde{x}^2 = x^2 + y^2$. The massless particle moves in the V direction with constant $U = U_0,\ \tilde{x} = 0$ and momentum p [9]. The effect of this metric on null geodesics is a discontinuity δV at $U = U_0$ [**?**]

$$\delta V(\tilde{x}) = -4p\ln(\frac{\tilde{x}^2}{M^2}) \tag{5}$$

Using the WKB approximation one can find [1] that up to an overall phase only one S-matrix agrees with Eq.(4)

$$\langle p_{out}(\tilde{x}^{'}) \mid p_{in}(\tilde{x})\rangle = N\exp\left(4i\int d^2\tilde{x}d^2\tilde{x}^{'}p_{out}(\tilde{x})f(x^{'},x)p_{in}(\tilde{x}^{'})\right), \tag{6}$$

where $p_{out}(\tilde{x}^{'})$ and $p_{in}(\tilde{x})$ are the momentum distributions of the out-state and in-state and $f(x^{'},x)$ is the Green function on the horizon $\ln(\frac{(\tilde{x}-\tilde{x}^{'})^2}{M^2})$. Notice that Eq.(6) is symmetric under time reversal. In fact the basic idea of quantum mechanics and hence of the S-matrix ansatz is to treat the out-state and in-state on an equal footing .[3] Since for ingoing light flux $T^{in}_{uu} = T^{in}_{uv} = 0$,

[3]Treating the in-state and outstate symmetrically is also the key in the weak value approach [3, 4].

time reversal implies that for outgoing radiation $T_{vv}^{rad} = T_{uv}^{rad} = 0$. Therefore, the S-matrix ansatz implies that

$$\begin{aligned} 4\pi r^2 T_{uu}^{rad} &= \frac{\alpha}{M^2} \\ 4\pi r^2 T_{vv}^{rad} &= 0 \\ 4\pi r^2 T_{uv}^{rad} &= 0 \end{aligned} \tag{7}$$

where α depends on the number of radiated fields and their spin [11]. We are now in a position to find the geodesics according to the S-matrix ansatz. We shall obtain the same result using two alternatives derivations. The first approach is to use the Rindler approximation to the Schwarzschild metric near the horizon,

$$ds^2 = dUdV + dX_i dX_i \tag{8}$$

Rindler and Schwarzschild coordinates are related by

$$\begin{aligned} \rho^2 &= UV \\ t &= 2M \ln(U/V) \end{aligned} \tag{9}$$

where ρ is the invariant distance from the horizon in a fixed Schwarzschild time

$$\rho = \int_{2M}^{r} dr \sqrt{g_{rr}} \approx \sqrt{8M(r - 2M)} \tag{10}$$

The effect of the gravitational shock wave of one outgoing Hawking particle at $V = V_0$ on an ingoing test particle is a discontinuity $\delta U \approx 1/V_0$. As a result ρ increases (see Figure 1). If there were only one Hawking particle then the test particle would have still cross the horizon (at a later time than it would

have in the absence of the outgoing particle). But, before the test particle crosses the horizon it crosses the shock waves of all the Hawking particles. By the time it has crossed all the shock waves the black hole has already evaporated completely so there is no mass left to form a black hole and a horizon. To obtain this conclusion in a rigorous way we can use Eqs.(4,7) and $u = 4M \ln(U/4M)$ to find that the back-reaction of Hawking flux on the metric near the horizon is

$$ds^2 = dUdV + \frac{a}{U^2}dU^2 + dX_i dX_i, \tag{11}$$

where $a = 16\alpha$. The null geodesics lines can be found for radial trajectories

$$\begin{aligned} U &= U_0 \\ V &= \frac{a}{U} + V_0 \end{aligned} \tag{12}$$

where V_0 and U_0 are positive constants. Therefore, the outgoing trajectories are the same as in Rindler space but the ingoing trajectories are such that

$$\rho^2 = UV = a + UV_0 \geq a \tag{13}$$

So an ingoing particle will eventually float at distance $\sqrt{a}$ above the horizon instead of crossing it.

The inertial coordinate system of the test particle can be found by making a coordinate change

$$\begin{aligned} V' &= V - \frac{a}{U} \\ U' &= U \end{aligned} \tag{14}$$

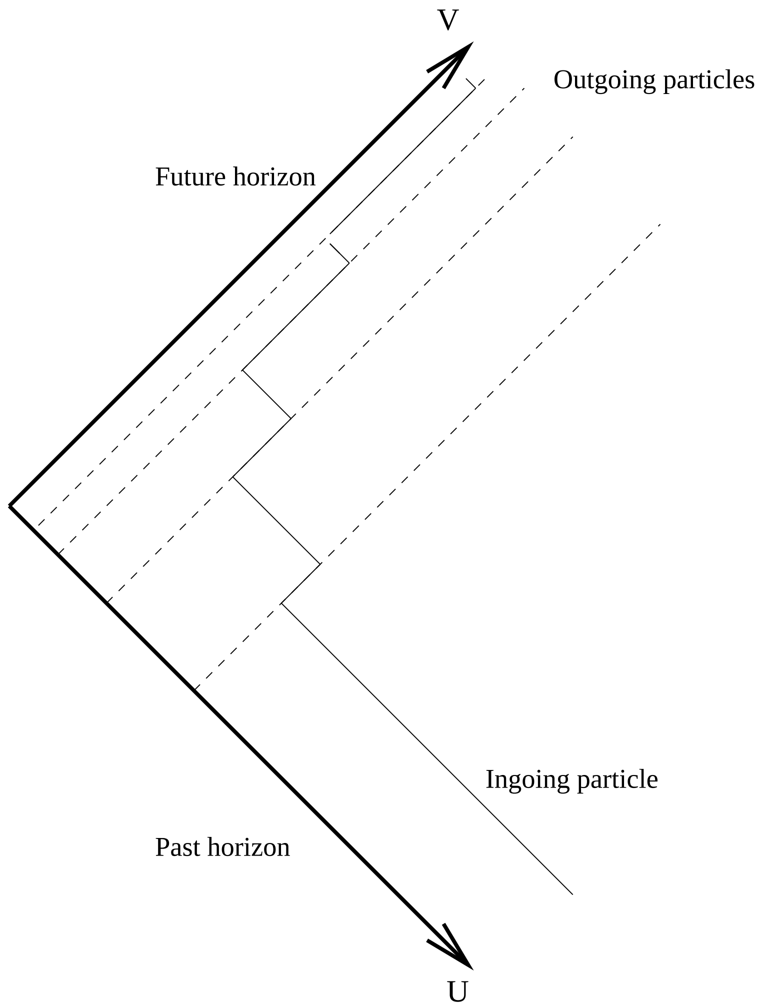

Figure 1: The effect of the shock wave of the Hawking radiation on an in going particle.

to get

$$ds^2 = dU^{'}dV^{'} + dX_i dX_i \tag{15}$$

The horizon in this metric is at $\rho^{'2} = V^{'}U^{'} = -a$ but this is a fictitious horizon since when the ingoing particle reaches $U^{'} = 0$ all Hawking particles were already emitted so there is no longer a black hole. We see therefore that the test particle which started at a flat space $\mathcal{I}^-$ will end up in flat space and not at the singularity (Figure 2a) though along the trajectory there is a region with large curvature.

An alternative way to reach the same result is to use the outgoing Eddington-Finkelstein coordinates. In the region where there is no infalling matter but only Hawking radiation Vaidya metric is an exact solution to Einstein's equation including Hawking radiation where $T^{rad}_{\mu\nu}$ is given according to the S-matrix ansatz (Eq.(7)).

$$ds^2 = -(1 - \frac{2M(u)}{r})du^2 - dudr + r^2 d\Omega^2, \tag{16}$$

where

$$M(u) = M - \int_{u_0}^{u} du T_{uu} \tag{17}$$

u_0 is the time at which Hawking radiation started. The ingoing radial null geodesic equation is

$$\frac{dr}{du} = -(1 - \frac{2M(u)}{r}) \tag{18}$$

Eventually the test particle will be near the horizon and it is helpful to define

$$\delta(u) = r(u) - 2M(u) \tag{19}$$

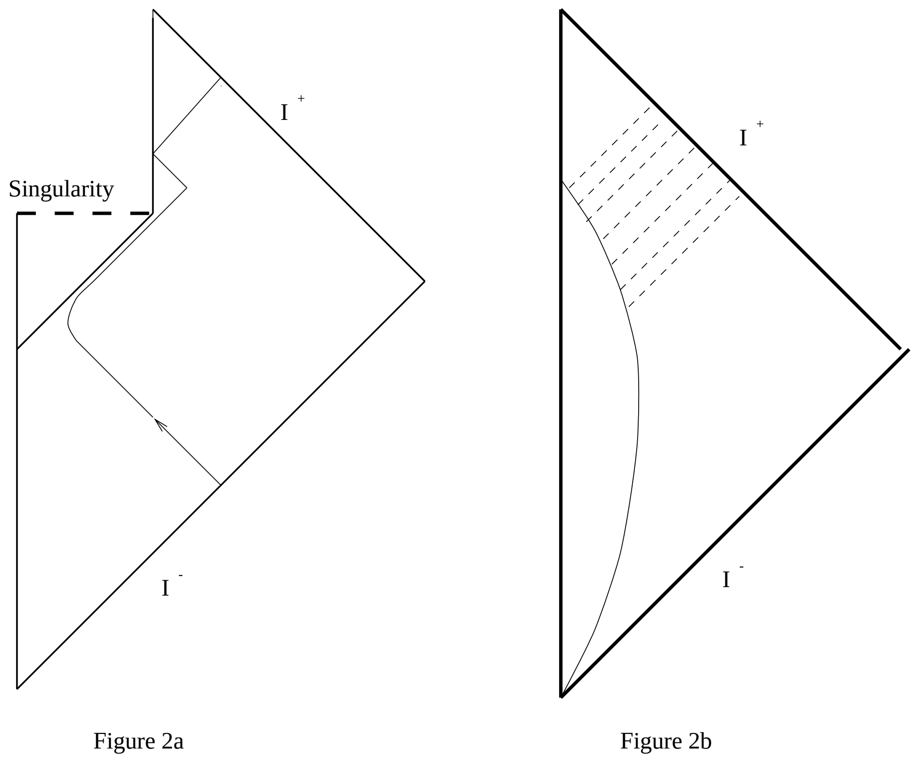

Figure 2: The trajectory of an ingoing light particle is such that it will not cross the horizon but will reach $\mathcal{I}^+$. Therefore, the Penrose diagram describing a collapsing star has a trivial topology.

to get

$$\frac{d\delta}{du} \approx \frac{\delta}{M} + 2\frac{dM}{du} = \frac{\delta}{M} - \frac{2\alpha}{M^2} \tag{20}$$

So asymptotically $\delta(u) = \frac{2\alpha}{M(u)}$. This means that the invariant distance from the horizon is $\rho = \sqrt{a}$ which is the same result as Eq.(14). Note that the exact form of $M(u)$ is not important to reach the conclusion that the test particle will not cross the horizon. The exact form of $M(u)$ is only important to find the invariant distance from the horizon at which the null geodesic will float. Therefore, fluctuations of T_{uu} cannot change the result that test particle cannot cross the horizon. Clearly, both Eq.(11) and Eq.(17) are not a good approximation to the metric in the region where there are ingoing particles. Nevertheless, the above discussion is a strong indication that the S-matrix ansatz implies such a back reaction that the Penrose diagram of a star which collapses to form a black hole has the same topology as Minkowski space (see Figure 2b).

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References

[1] G. 't Hooft, Nucl. Phys. B335 (1990) 138.

[2] L. Susskind, L Thorlacius and J. Uglum, Phys. Rev. D48 (1993) 3743.

[3] C. R. Stephens, G. 't Hooft and B. F. Whiting, Class. Quan. Grav. 11 (1994) 621.

[4] F. Englert, "Operator Weak Values and Black Hole Complementarity" ULB-Th 03/95 gr-qc/9502039 in proceeding of the Oskar Klein Centanary Symposium, edited by U. Lundstroem (World Scientific, Singapore, 1994).

[5] Y. Aharonov and L. Vaidman, J. Phys. A 24 (1991) 2315.

[6] P. C. W. Davies, S. Fulling and W. G. Unruh Phys. Rev. D 13 (1976).

[7] S.W. Hawking, Comm. Math. Phys. 43 (1975) 199.

[8] G. 't Hooft, Nucl. Phys. B256, (1985) 727.

[9] P.C. Aichelburg and R.U. Sexl, J. Gen. Rel. Grav. 2 (1971) 303.

[10] R. Brout, S. Massar, R. Parentani and Ph. Spindel, Phys. Rept. 260 (1995) 329.

[11] D. N. Page Phys. Rev. D13 (1976) 198.